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The materials presented can be used in further scientific and academic process at universities and colleges, are of interest both for the general reader and professionals.

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STATISTICAL METHODS OF SIGNAL AND DATA PROCESSING (SMSDP-2010)

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ORGANIZED

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OBJECTIVES AND GROUNDS

The Conference is planned to be a forum of ideas exchange and results discussion in the field of methods, algorithms and means of data and signal processing. Submissions that contain novel results is signal detection, parameter estimation, and other kind of statistical processing of signals and data in telecommunication, remote sensing and information systems are welcome.

SESSIONS

1. Statistical models of signals and fields.
2. Detection of signals. Signal and interference parameters estimation.
3. Signal and data processing in remote sensing.
4. Discrete signals processing in telecommunication systems.
5. Data and signal processing in information systems.

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Comparison of statistical parameters of real and simulated electroencephalographic signals

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Abstract — **Statistical parameters of real and simulated electroencephalogram (EEG) are important for testing the statistical methods of signal analysis. The better is the model the more it satisfies the requirements not only in time-frequency domain, but the requirements of proximity to the statistical parameters of real EEG signal. Evaluation of adequacy and applicability of one of the existing EEG model using statistical parameters is realized. Preferable statistical parameters of EEG which are preserved by the model under consideration are selected. The test results proving the parameter selection are presented.**

Keywords - EEG processing; EEG simulating; Statistical properties of EEG

I. INTRODUCTION

Electroencephalography is one of the most widespread techniques for evaluating the state of human brain. It represents the electrical activity of neurons in the brain and gives outstanding possibility to visualize the functioning of the brain in real time. During the electroencephalographic test the multichannel signal of voltage differences from the surface of the head – electroencephalogram (EEG) is recorded and further analyzed. The amplitude, time, frequency, statistical and scale parameters of EEG signal are examined by the physician. They are useful for diagnostics of epilepsy and other brain diseases as well as for determining the parameters of normal brain functioning [1, 2].

Today modern computerized EEG recording devices allow to use versatile powerful mathematical techniques for obtaining various parameters of EEG to derive new information about brain functioning. Thus many new sophisticated methods of EEG analysis are being developing to enhance the possibilities of diagnostics. The final stage for evaluating the performance of any new signal processing technique before using it in the real-life applications is its testing with EEG signals with different properties. Often there is absence of real EEG signals with all needed properties for comprehensive assessment of newly developed techniques. In this case the use of simulated EEG signals could be of great advantage [3].

The crucial problem in using the simulated signals instead of real ones is the assurance that degree of conformity between real and simulated signal is sufficiently high. In opposite case the use of simulated signal for testing the method's performance is meaningless. Thus for each signal model a comparison of similarity between real and simulated signals should be done beforehand. There are still no general recommendations in the literature for using some particular parameters for such analysis of signal model.

It is known that the EEG signal could be considered as stochastic process, so it makes sense to define its statistical parameters. In this paper the task of comparison of real and simulated EEG signals using statistical parameters is considered. EEG model used in this work is based on the inverse Fourier transform and predefined averaged power spectral density for the rhythms [3]. The paper is organized as follows. First, brief information about the EEG model is given. Second, the possible set of parameters for comparison is selected and validated. In the last section the experimental results are presented and conclusions about the considered EEG model are given.

II. MODEL OF EEG SIGNAL

There are many ways of simulating EEG signals, such as generating an EEG signal by modeling activity of neurons or as random signal with the desired statistical characteristics etc. In the previous work [3] the method of simulation of EEG signal using a superposition of harmonic oscillations was proposed, using which simulated EEG signal could be obtained as:

$$y(t)=\sum_{n=1}^{M} A_n \sin(2\pi f_n+\varphi_n),$$

where A_n – amplitudes, f_n – frequencies, φ_n – phase shift for each sinusoidal curve; M – number of harmonic oscillations used and length of vectors A_n and f_n. By manipulating the vector of amplitudes the desired signal spectrum can be obtained and the challenge is to calculate the amplitude of each sinusoid of a certain frequency. The new technique for selecting the averaged power spectral density for each EEG rhythm in the simulated signal was also presented.

This method allows prescribing the power density spectrum of simulated signal and it is capable to reflect time-frequency parameters of real EEG for different states of brain in the simulated signals.

III. Statistical Parameters of Electroencephalogram

It is widely known that signals could be analyzed with different mathematical approaches: deterministic and statistical. Each of them has its own merits and demerits, and reveals different properties of the signal under consideration. The choice of particular methods depends on the current needs of researcher and current task.

The proposed model of EEG signal, briefly described in previous section implies that simulated signal has no random parameters except phase of harmonics. Thus the deterministic parameters of obtained signal's model are in general already known. On the other hand, many researchers emphasized that brain electrical activity has considerable amount of randomness, representing the stochastic way of information processing in the brain structures [1, 2]. Thus evaluation of statistical parameters for simulated EEG signal and comparing these parameters with such of real EEG is useful for evaluating the quality of proposed model.

In this work we consider EEG signals as discrete stationary random functions of time. Let we have the sample of EEG signal of length N: $X(t_i)=x_i$, $i=\overline{1,N}$. As measures of similarity between real and simulated EEG signals the following parameters were selected [4-6]:

1. Averaged standard deviation for signals.
2. Central and ordinary moments of EEG realizations.
3. Normalized histogram.
4. Skewness.
5. Kurtosis.

Sample estimate of *ordinary moments* of k-th order is

$$m_k=\frac{1}{N}\sum_{i=1}^{N}(x_i)^k. \tag{1}$$

Central moments of different order completely describe the random process. The sample central moment of order k is defined as

$$\mu_k=\frac{1}{N}\sum_{i=1}^{N}\left(x_i-m_1\right)^k, \tag{2}$$

where $m_1=\frac{1}{N}\sum_{i=1}^{N}x_i$ – estimate of first order ordinary moment (sample mean).

Histogram is the sample estimate of probability density function of random process. First we divide real axes into $K=1+3.32\cdot\lg N$ parts with equally spaced limits: $-\infty<v_1<v_2<\ldots<v_K<+\infty$ and count the number h_j of samples of signal $X(t_i)$ which occurred in particular j-th interval, $j=\overline{1,K}$. Then step function is called histogram:

$$H(x_j)=h_j,\ x_i\in\left(v_{j-1},v_j\right],\quad j=\overline{1,K},\ i=\overline{1,N} \tag{3}$$

Normalized histogram which is the estimation of sample probability density function, is defined as

$$H(x_j)=\frac{h_j}{N\Delta v_j},\ x_i\in\left(v_{j-1},v_j\right],\quad j=\overline{1,K},\ i=\overline{1,N}, \tag{4}$$

where $\Delta v_j=v_{j-1}-v_j$.

Skewness is the measure of asymmetry of random samples scattering around the mean value, which is the estimate of asymmetry of probability distribution function. If skewness is negative the data are more distributed to the left from the mean value, and in opposite case the data are spread out more to the right side from the mean value. Skewness is defined as

$$S=\frac{\mu_3}{\sigma^3}, \tag{5}$$

where μ_3 is third order sample central moment and σ is sample standard deviation. For discrete random function, skewness is defined as

$$S=\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-m_1)^3}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i-m_1)^2\right)^{3/2}}. \tag{6}$$

Skewness is extremely useful in the cases when the data considered normally distributed are in real life differently deviated around the mean value. Thus employing skewness can give additional important information about the random signal properties.

Kurtosis is the estimate of sharpness of the probability distribution's peak; it shows how plain the distribution is and thus could be indirect measure of the value of variance. Sample kurtosis is defined as

$$\gamma=\frac{\mu_4}{\sigma_4}-3=\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-m_1)^4}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i-m_1)^2\right)^{4/2}}-3. \tag{7}$$

The term -3 is included to assure the kurtosis of normal distribution equal to zero.

IV. Experiments

During the experiments the abovementioned parameters were calculated for real and simulated EEG. Real EEG signal were recorded in the Department of Neurophysiology of Institute of Neurosurgery of Ukraine using the Galileo Planet 200 computerized EEG recorder. All calculations were performed in the Department of Physical and Biomedical Electronics of National Technical University of Ukraine "Kyiv

Polytechnic Institute" in Matlab environment. To calculate the statistical parameters 19 EEG signals were used for each case. Real and simulated EEG signals for healthy person were calculated using model described in Section II. Corresponding normalized histograms are presented on Fig. 1, 2. Also the real and simulated EEG signals for person suffering from epilepsy were used; corresponding normalized histograms are presented on Fig. 3, 4. For the cases of EEG of healthy person and person suffering from epilepsy the following parameters were calculated (1-7): averaged standard deviation, ordinary and central moments of order 1 to 5, skewness and kurtosis. Results are given in Tables I, II.

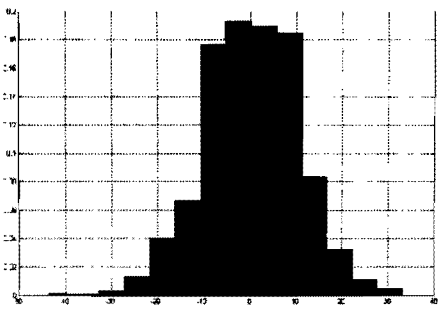

Figure 1. Normalized histogram for real EEG signal of healthy person

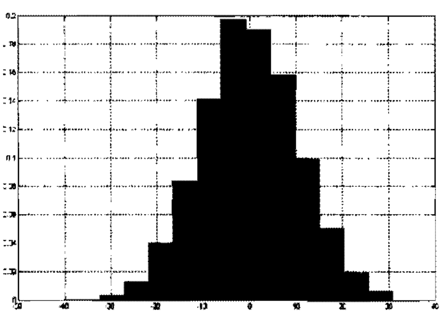

Figure 2. Normalized histogram for simulated EEG signal of healthy person

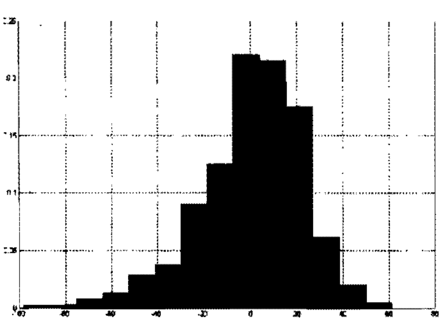

Figure 3. Normalized histogram of real EEG signal of person suffering from epilepsy

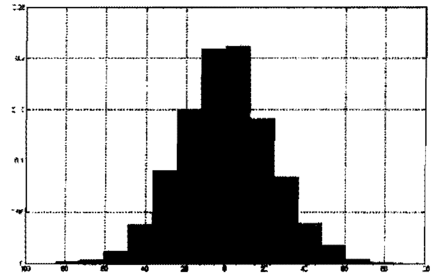

Figure 4. Normalized histogram of simulated EEG signal of person suffering from epilepsy

TABLE I. PARAMETERS OF REAL AND SIMULATED SIGNALS OF HEALTHY PERSON

Parameter, signal from healthy person	**Real EEG**	**Simulated EEG**
Averaged Standard Deviation	9.88 ± 1.44	10.37 ± 5.63e-014
1st Ordinary Moment	0.52 ± 0.32	1.48e-015 ± 5.04e-015
2nd Ordinary Moment	100.14 ± 27.82	107.60 ± 1.16e-012
3rd Ordinary Moment	169.11 ± 639.51	-15.90 ± 51.47
4th Ordinary Moment	47410.76 ± 59810.93	34931.39 ± 848.54
5th Ordinary Moment	1346607.58 ± 6291420.08	-21790.84 ± 65412.05
1st Central Moment	0 ± 0	0 ± 0
2nd Central Moment	99.77 ± 27.82	107.60 ± 1.16e-012
3rd Central Moment	16.07 ± 648.23	-15.90 ± 51.47
4th Central Moment	47292.39 ± 58836.42	34931.39 ± 848.54
5th Central Moment	1224500.76 ± 6161195.26	-21790.84 ± 65412.05
Kurtosis	4.19 ± 4.13	3.01 ± 0.07
Skewness	-0.003 ± 0.53	-0.01 ± 0.04

V. DISCUSSION

As can be seen from Tables I and II, some values are quite diverse for same parameters for real and simulated EEG signal for healthy and sick person and others are very close. Thus the precise analysis of each parameter's applicability for using in signal analysis is needed as well as comparison of values for real and simulated signals.

Averaged standard deviation for real and simulated signals for both cases is the same, but for EEG of healthy person it equals nearly 10, while for the case of EEG of sick person it is approximately twice larger and equals nearly 20. Thus considered EEG model preserves the standard deviation of real EEG in simulated signal and can be used in the cases when this is meaningful characteristic of signal. Averaged standard deviation could be useful characteristic for classifying EEG signals.

TABLE II. PARAMETERS OF REAL AND SIMULATED SIGNALS OF SICK PERSON

Parameter, signal from epileptic person	Real EEG	Simulated EEG
Averaged Standard Deviation	19.55 ± 2.72	22.41 ± 1.14e-013
1st Ordinary Moment	0.58 ± 0.49	1.73e-015 ± 1.17e-014
2nd Ordinary Moment	390.14 ± 104.35	502.35 ± 5.06e-012
3rd Ordinary Moment	255.23 ± 2167.16	-116.92 ± 575.63
4th Ordinary Moment	537633.1 ± 260975.19	759870.51 ± 27396.85
5th Ordinary Moment	-641439.63 ± 10266887.78	-727535.52 ± 3521040.41
1st Central Moment	0±0	0±0
2nd Central Moment	389.57 ± 104.3	502.35 ± 5.11e-012
3rd Central Moment	-433.28 ± 1886.23	-116.92 ± 575.63
4th Central Moment	536231.55 ± 260222.3	759870.51 ± 27396.85
5th Central Moment	-2218358.09 ± 9789379.59	-727535.52 ± 3521040.41
Kurtosis	3.35 ± 0.36	3.01 ± 0.11
Skewness	-0.05 ± 0.22	-0.01 ± 0.05

The values of EEG moments of all orders except 2nd differ for real and simulated EEG for both cases. Moreover, the study showed that standard deviation of ordinary and central moments is often much larger then mean values. To investigate this fact and find an explanation further research is needed, for now we can conclude that EEG moments should be utilized with discretion for EEG analysis.

From visual analysis of histograms obtained for all signals it can be concluded that the model of EEG signal does not allow to completely resemble all properties of signal's probability density function (PDF). It is confirmed by comparison of skewness for real and simulated signal. For healthy person mean skewness for simulated EEG is four times less than for real signal (0.003 vs. 0.01). For EEG of sick person the real signal's mean skewness is five times larger than for simulated signal (0.05 vs. 0.01). Such difference is high but more important is the fact that differences for different signal types are opposite. Thus we can conclude that EEG model does not preserve the skewness and it is needed to use simulated EEGs carefully in the cases when distribution of signal's values around the mean is important. For example this could be the case of investigation of exceeding of brain activity over the isoline or over averaged value of brain electrical activity when EEG is registered by the weighted reference scheme.

On the other hand the values of kurtosis for all signals could be considered as almost the same, which proves the applicability of the EEG model for the cases when the sharpness of PDF is substantial. For example kurtosis can be useful measure for investigation of small-magnitude brain waves distribution.

Also it can be seen that the obvious way to improve the simulation quality is to include the possibility to set the mean value of desired EEG signal into model as constant to add to the sum of sinusoidal waves (1).

VI. CONCLUSIONS

The recently developed model of EEG signal is analyzed for capability to represent statistical properties of real signal for the cases of EEG of healthy and sick person. Averaged standard deviation, central and ordinary moments up to 5th order, skewness and kurtosis was utilized as possible characteristics of EEG. In the result two of signal's parameters could be recommended as reflected by the model adequately, such as standard deviation and kurtosis. These parameters were the same for real and simulated EEG signals.

To additionally prove similarity of signals and adequacy of the model subsequent work will be devoted to further evaluation of EEG simulation quality and in particular to estimation of possible nonstationarity of obtained EEGs. It is also of great interest to add more flexibility to existing model to represent different brain states in EEG signals.

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