SIGNAL PROSSESING SYMPOSIUM

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Warsaw University of Technology – Institute of Electronic Systems

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Welcome Message
from the Chairman
of Signal Processing Symposium

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It is my great pleasure to welcome you in Poland at the beautiful artificial lake “Zalew Zegrzynski” in Jachranka village. On behalf of the Programme Committee of this conference it is my great pleasure to welcome everybody very cordially to the Signal Processing Symposium SPS- 2009. That symposium is a part of the 24th IEEE-SPIE Symposium Photonics, Electronics and Web Engineering.

The conference is organized by Institute of Electronic Systems, Warsaw University of Technology together with Przemyslowy Instytut Telekomunikacji S.A., Military University of Technology, RADWAR S.A, Space Research Center of Polish Academy of Sciences, Foundation for Development of Radiocommunication and Multimedia Technologies, IEEE Signal Processing Society Poland Chapter and IEEE AES Poland Chapter

With the Signal Processing Symposium SPS- 2009, the Warsaw Technical University, Institute of Electronic Systems continues to organize the series of very successful signal processing symposia which started in 2003 in Wilga followed by SPS-2005 also in Wilga in 2003 and SPS-2007 in Jachranka in 2007.

The first Signal processing symposium SPS-2003 was a part of the 12th IEEE-SPIE Symposium on Advanced Electronics. It gathered more than 30 participants and 4 guests from abroad. Nineteen high level articles were presented at oral and poster sessions. In 2005 me and my students from The Radiolocation and Digital Signal Processing Students' Research Group decided to organize the second edition of that symposium - Signal Processing Symposium SPS2009, which was a part of the 16th IEEE-SPIE Symposium on Photonics, Electronics and Web Engineering. That conference gathered more than 100 participants from 4 countries. More than sixty high level articles were presented at oral and poster sessions. The next one, SPS-2007 gathered also more than 100 participants from 6 countries.

The main goal of that symposium is to create forum for students and scientists to present their latest research results, new trends in science and technology and exchange ideas during technical and evening sessions. I hope that all participants will enjoy their stay in Jahranka.

Krzysztof Kulpa
Chairman of the Signal Processing Symposium

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Computation of continuous wavelet transform of discrete signals with adapted mother functions

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ABSTRACT

Problems concerning the calculation of continuous wavelet transform are considered for the case when adapted mother wavelet is specified only by its discrete values. The new technique of wavelet coefficients' computation for arbitrary scales is proposed. Experimental results of identifying of the epileptiform complexes in the test human electroencephalogram have proven the usefulness of developed technique.

Keywords: continuous wavelet transform, discrete signals, arbitrary scales, EEG processing, epileptiform complexes

INTRODUCTION

Wavelet analysis has become a standard tool for parsing time-frequency behaviour of signals [1,2]. One of the urgent problems which are being solved using continuous wavelet transform (CWT) of discrete signals is time analysis of signals for the purpose of identifying the underlying signal patterns. Unlike the classic methods of Fourier analysis, which represent signals as a complex harmonic spectrum, wavelet-analysis gives mediated information about frequency structure of signal and its changing history. For instance, this task is appearing during the search of epileptic oscillations' complexes in human electroencephalogram. For the identification of such complexes one can use an adapted mother wavelet function that has the same time properties like the patterns of interest in the signal. Because of the nature of epileptic complexes generation, their duration will not significantly change relative to some standard known complex duration. Herewith, mother wavelet should not be stretched and squeezed considerably, but because of the need to identify even small changes in possible complex duration, the values of scales could be arbitrary real numbers. Hence, scale coefficients during CWT ought to change in relatively small bounds around one. For decomposition coefficients obtaining in the course of arbitrary scales isn't possible to employ known methods of multiresolution analysis, which imply only the dyadic scales change.

Authors know only one work, where the method of computation of CWT coefficients of discrete signals is proposed for arbitrary scales, which is based on approximation of signal under investigation and wavelet function by basis splines [3]. Thus in the literature to the problem of continuous wavelet analysis of discrete signals with the use of the adapted discrete mother functions, specified only by the discrete values and for the case of arbitrary scales, is not spared enough attention.

In this work the new technique of calculation of CWT coefficients of the discrete signal for arbitrary scales is proposed. It was considered a special case when the mother wavelet function is specified only by its discrete values in the time moments corresponding to the signal samples and has not definite mathematical expression, therefore its stretched and squeezed form's values can't be explicitly calculated. Such case may appear when the mother wavelet is obtained using the adaptive algorithm according to the properties of electroencephalogram signal parts to be identified [4].

1. CONTINUOUS WAVELET TRANSFORM OF DISCRETE SIGNALS

The detailed theory of wavelet transforms is thoroughly presented in many sources; classic textbooks are [1, 2]. In this work we briefly highlight only substantive aspects. Let $\psi(t)$ be continuous function, which satisfies the "admissibility condition" $C_\psi = \int_0^{+\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{\omega} d\omega < +\infty$ and has unit norm $\|\psi(t)\| = \sqrt{\int_{-\infty}^{\infty} |\psi(t)|^2 dt} = 1$, which is called "mother wavelet function". Implementation of the admissibility condition in time domain is equivalent to the requirement that the mother

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wavelet must have zero mean: $\int_{-\infty}^{+\infty} \psi(t)\,dt = 0$. For the predefined scaling coefficients $a \in \mathbb{R}, a \neq 0$ and shifts $b \in \mathbb{R}$ one can obtain the set of scaled (stretched or compressed) and shifted mother wavelets:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right). \tag{1}$$

For the scales a and translations b the **continuous wavelet transform of continuous signal** $s(t)$ of infinite duration with mother function $\psi(t)$ is:

$$W_\psi(a,b) = \langle s(t), \psi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(t) \cdot \psi^*\left(\frac{t-b}{a}\right) dt\ , \tag{2}$$

where $\psi^*(t)$ is the complex conjugate of $\psi(t)$ and $\langle \bullet, \bullet \rangle$ denotes inner product.

According to the theorem of Calderon, Grosmann and Morle [1], if the admissibility condition is held, then the CWT is energy-preserving transform, the inverse continuous wavelet transform exists, and any signal $s(t)$ satisfies

$$s(t) = \frac{1}{C_\psi} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} W_\psi(a,b) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) db \frac{da}{a^2},$$

$$\int_{-\infty}^{+\infty} |s(t)|^2\, dt = \frac{1}{C_\psi} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} |W_\psi(a,b)|^2\, db \frac{da}{a^2}.$$

Because virtually all signals under investigation are digitized versions of continuous real signals, which appear to be in the form of sequence of numbers, modification of CWT for the case of discrete signals is needed. Let us consider the modification of CWT (2) for the case when the continuous signal $s(t)$ is subject to uniform sampling. In order to take advantage of the expression for CWT of continuous signals, we can construct continuous signal using the available samples and the Dirac delta function:

$$s_\delta(t) = s(nT_s) = \sum_{n=-\infty}^{+\infty} s(t)\delta(t - nT_s),$$

where $s_\delta(t)$ is the continuous signal, taking on non-zero values equal to the samples of $s(t)$ only in the time instants of sampling. Substituting this signal into (2) will give:

$$\begin{aligned} W_\psi(a,b) &= \langle s_\delta(t), \psi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s_\delta(t) \cdot \psi^*\left(\frac{t-b}{a}\right) dt = \\ &= \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \left(\sum_{n=-\infty}^{+\infty} s(t) \cdot \delta(t - nT_s) \right) \cdot \psi^*\left(\frac{t-b}{a}\right) dt = \\ &= \frac{1}{\sqrt{a}} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} s(t) \cdot \delta(t - nT_s) \cdot \psi^*\left(\frac{t-b}{a}\right) dt. \end{aligned}$$

For the sake of simplicity we can put $T_s = 1$ without loss of generality and thus restrict ourselves to only integer time shifts $m \in \mathbb{Z}^+$. Taking into account the filtering property of Dirac delta function we can finally write more accurate expression for CWT of discrete signal for the case when signal has duration M samples:

$$W_{\psi}\left(a,m\right)_{\delta}=\frac{1}{\sqrt{a}}\sum_{n=0}^{M-1}s\left(n\right)\cdot\psi^{*}\left(\frac{n-m}{a}\right). \quad (3)$$

In this expression should be emphasized the ability of using not only dyadic but arbitrary scaling factors *a* that is of great importance while employing the CWT for time-scale analysis of discrete signals.

2. CWT OF DISCRETE SIGNALS FOR ARBITRARY SCALES AND DISCRETE MOTHER FUNCTION

It is seen from (3), to calculate the CWT of the discrete signal $s\left(n\right)$, the samples of scaled mother wavelet function $\psi\left(\frac{n-m}{a}\right)$ in the time instants for given signal samples are needed. For the case when there is an explicit equation for mother function, this problem is trivial. It can be solved in general case by simple calculating the values of scaled mother function and substituting them in the expression of discrete inner product (3). But when there is no exact mathematical expression for mother function, this challenging task of evaluating CWT coefficients still has a lack of solution. This is often the case of discrete adapted mother wavelets, which are specified by only the discrete values in the sample points [4]. One of the major difficulties consists in finding of the discrete values of stretched and squeezed wavelets for arbitrary scales *a*. This paper is focused on the development of the technique for dealing with such cases.

One of the possible solutions of the problem is to use the auxiliary grid of sample knots and subsequent interpolation. Let us consider the set of scales $A=\left\{a_1,a_2,...,a_L\right\}$, $a_j>0$, $j=\overline{1,L}$, needed for the CWT of the signal, and let us have the discrete mother wavelet function, specified in *N* points $\psi\left(n\right)$, $n=\overline{0,N-1}$. For every $a_j\in\left(0,1\right)$ the scaled wavelet function will be squeezed relative to mother function and will have less samples, and for the scales $a_j\in\left(1,+\infty\right)$ scaled wavelet will be stretched and have more samples. But all scaled functions must be specified on the same grid as the signal under consideration. The following sequence of steps for obtaining the CWT coefficients for the case is proposed.

1. For every scale $a_j\in A$ evaluate the number of samples of scaled wavelet: $K_j=\left[a_jN\right]$, where square brackets stand for the integer part.

2. Generate the regular grid with points k_i between 0 and *N* with spacing $\frac{N}{K_j}$.

3. Having the discrete values of mother wavelet $\psi\left(n\right)=\left\{\psi\left(0\right),\psi\left(1\right),...,\psi\left(N-1\right)\right\}$, approximate the values of the scaled wavelet function $\tilde{\psi}\left(n\right)$ in the knots k_i, thus getting K_j values of scaled function on the grid k_i, $i=\overline{1,K_j}$ between 0 and *N*. For the current task we propose to use cubic spline interpolation for obtaining the approximate values of the wavelet functions on the new grids.

4. Return to initial grid and compose the scaled wavelet $\tilde{\psi}_j\left(n\right)=\tilde{\psi}\left(k_i\right)$, $n=\overline{0,K_j-1}$, $i=\overline{1,K_j}$ with spacing equal to 1; set $\psi_j\left(n\right)=0$ for $n=\overline{K_j,N-1}$. Thus for every scaling coefficient a_j the discrete values of scaled mother wavelet function could be obtained using only the discrete values of mother wavelet without explicit mathematical expression.

5. Following the restrictions, in which the wavelet is constrained, namely, equality of the norm to one and the admissibility condition, lead for our case to the equations for discrete mother functions:

$$\left\|\psi(n)\right\| = \sqrt{\sum_{n=0}^{N-1}\left|\psi(n)\right|^2} = 1,$$
$$\sum_{n=0}^{N-1}\psi(n) = 0. \qquad (4)$$

For CWT of discrete signals implemented for adapted discrete mother functions it is necessary to fulfill conditions (4) as well. To ensure this for scaled wavelets $\psi_j(n) = \frac{1}{\sqrt{a_j}}\psi\left(\frac{n}{a_j}\right)$, normalization of every scaled function after spline interpolation is proposed. Thus instead of $\tilde{\psi}_j(n)$ its normalized version is obtained using expression

$$\tilde{\psi}_j(n) \Rightarrow \frac{\tilde{\psi}_j(n)}{\left\|\tilde{\psi}_j(n)\right\|} = \frac{\tilde{\psi}_j(n)}{\sqrt{\sum_{n=0}^{N-1}\left|\tilde{\psi}_j(n)\right|^2}}$$

should be employed.

6. Substitute in (3) new function $\tilde{\psi}_j(n)$ instead of $\psi(n)$ thus having after evaluating of complex conjugate final expression:

$$W_\psi\left(a_j, m\right)_\delta = \frac{1}{\sqrt{a_j}}\sum_{n=0}^{M-1} s(n)\cdot\tilde{\psi}_j^*(n-m), \quad j = \overline{1, L}\,. \qquad (5)$$

It was taken into account in (5) that scaling and shifting are independent operations, and one could at first evaluate all values of scaled wavelets and then perform shifting of the wavelets along the signal with arbitrary step *m*.

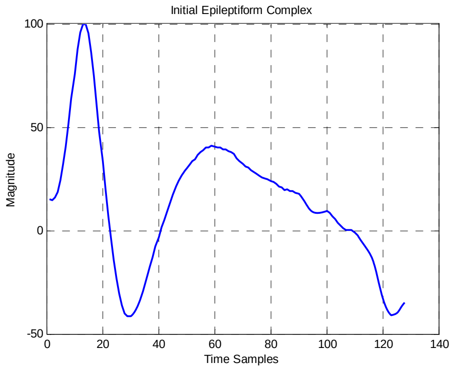
Thus the technique for estimation of discrete values of scaled copies of mother wavelet in the sample moments after stretching and squeezing corresponding to arbitrary scale was developed and presented above by the steps 1 – 6. Generally, the feature of the proposed technique is the precalculation of the values of all scaled discrete wavelets without explicit mathematical expression for all needed scales and subsequent pointwise evaluating the CWT coefficients of the signal of interest for any required time shift.

3. EXPERIMENTAL RESULTS

The task of experimental part of this work was the verification of the proposed technique by identification the epileptiform oscillations' complexes in a test electroencephalogram (EEG) using the wavelet coefficients values, obtained with adapted discrete mother function. An electroencephalogram is a record of multichannel signal of voltage differences, which can be the registered on an intact surface of the head. This signal is the result of superposition and spatial filtration in brain tissue of elementary processes which occur in all neurons during brain functioning. Many researchers consider EEG a noise-like process which represents total functional activity of whole brain. EEG shows oscillations with the amplitude of from 5 to 500 uV (normally – about 20-50 uV). As a rule, for routine procedures the spectrum of EEG is limited from 1 Hz to 30-40 Hz.

One of the vital problems which can be solved using continuous wavelet transform in electroencephalography for diagnostics of epilepsies is the search of epileptic oscillations' complexes in human multichannel electroencephalogram. Such complexes are inherent for people suffering epilepsy. Very often they have the appearance of complexes of a sharp wave and a slow wave. A presence of epileptiform complexes of different magnitude and duration in the EEG signal can be the of passing of hypersynchronous in the large groups of neurons in the brain, that makes their finding especially valuable for research and diagnostics of epilepsy.

Experimental studies were performed using MATLAB. Test signal of EEG was generated using the white Gaussian noise with zero mean value and unit variance with the sampling frequency 256 Hz. In order to fit to the real signal, which is used in routine investigations, the noise signal was filtered using the 9-th order Butterworth band pass filter with cutoff frequencies 5 and 30 Hz. For the test signal of EEG with the epileptiform complexes of different duration, real epileptiform complex of 0.5 seconds duration and amplitude of 100 uV (Fig. 1a) was selected from an existent database. Since it is known that duration of epileptiform complexes in the real EEG signal can make mainly from 0.1 to 1.2 seconds, using the existing real complex the stretched (up to 2.4 times) and compressed (up to 0.2 times) on duration complexes were built and inserted in the test signal with 3 seconds interval (Fig. 1b).



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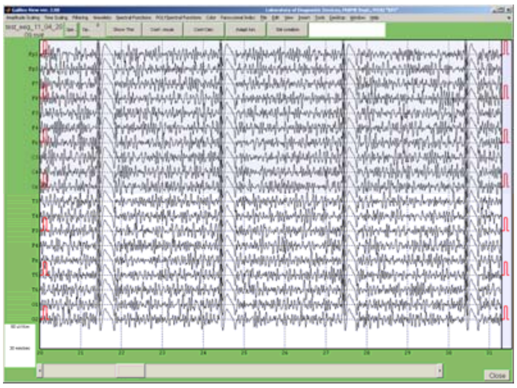
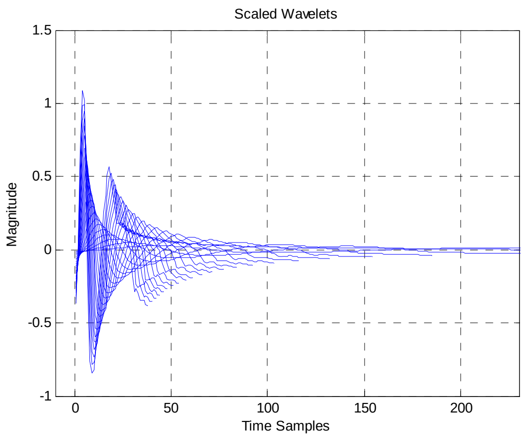


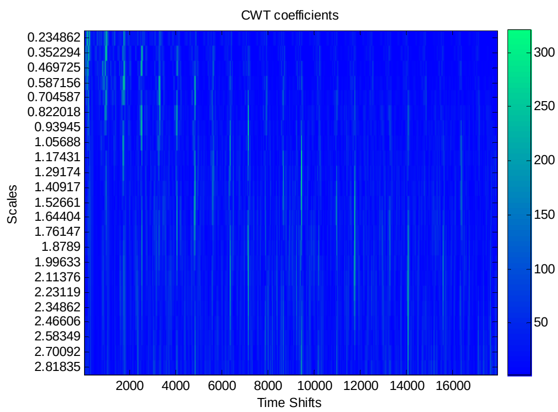
b

Figure 1. a – initial real epileptiform complex; b – test EEG signal with inserted epileptiform oscillations complexes

Using the existing technique of constructing the adapted mother wavelets on the basis of the main eigenvector of the matrix of averaged correlations [4], the samples of mother wavelet function which is adapted to the epileptiform complexes, were evaluated. Duration of the adapted mother function appeared to be 109 samples or 0.4258 s. Taking into account that the durations of real epileptiform complexes are known, the necessary scales of continuous wavelet transform were in further computations. The scales for obtaining the needed duration of real complex from the adapted mother function may vary from 0.2349 to 2.8183. The scaled discrete wavelets obtained using the proposed technique, are shown on the Figure 2a. Scalogram of the test EEG signal with inserted epileptiform complex (Fig. 1b) is presented on Fig. 2b. The bright regions on the scalogram correspond to the presence of epileptiform complexes of the signal.



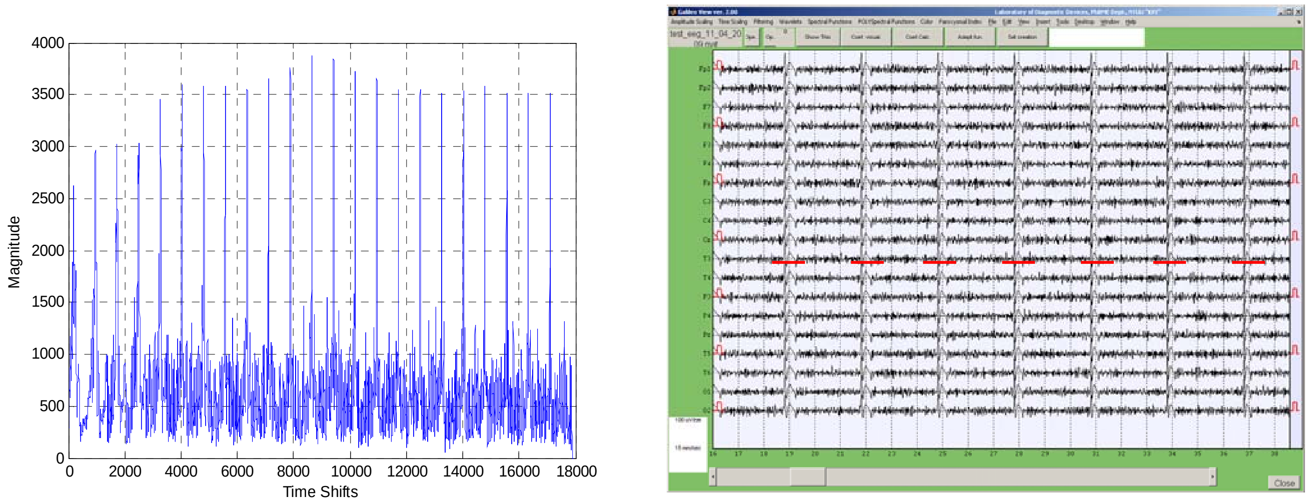
a



b

Figure 2. a – scaled adapted discrete wavelet functions; b – scalogram of the test EEG

The CWT coefficient can be regarded as the measure that represents the correlation between scaled mother function and signal segment under consideration. As the scales of mother wavelet which correspond to the duration of real complexes are known, the sum of CWT coefficients for appropriately selected scale range was chosen as the measure of the occurrence of epileptiform complexes in the signal (Fig. 3a).



a

b

Figure 3. a – the sum of CWT coefficients for the scales *a* from 0.2349 to 2.8183; b – test EEG signal from Fig. 2b with found epileptiform complexes underlined

It can be seen from the Fig. 3, the peaks in the CWT coefficients correspond to the presence of scaled epileptiform complex in the EEG. The segments of EEG with inserted complexes are reproduced as the sharp peaks on the plot of the sum of CWT coefficients, obtained using the technique developed in this paper. Thus, the experimental results of using the proposed technique show good efficiency for continuous wavelet analysis of test electroencephalogram for identifying the complexes of epileptiform oscillations.

CONCLUSIONS

The technique of calculation of continuous wavelet transform of discrete signals is proposed, enabling computation of the CWT in case when the mother wavelet function is specified only by its discrete values in the time moments corresponding to the signal samples and hasn't definite mathematical expression, therefore its stretched and squeezed form values can't be explicitly calculated. For such case the technique for estimation of discrete values of scaled copies of mother wavelet in the sample moments after stretching and squeezing corresponding to arbitrary scale was developed. Experimental results of using the proposed technique showed good efficiency for continuous wavelet analysis of the test electroencephalogram for detecting the epileptiform complexes.

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