INTRODUCTION

Dynamic response is an important characteristic of spirometers for testing of lung ventilation function (LVF) by measuring airflow rate during breathing. Since first standard issued by American Thoracic Society (ATS) in 1979 [1], this characteristic is standardized in all subsequent standards of ATS, European Respiratory Society (ERS) and spirometry methodological recommendations of Russian Federation (RF) [2–7] (Table I).
Table 1.

<table>
<thead>
<tr>
<th>Requirements for the frequency response nonlinearity in various standards</th>
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<td>±10% in range 0 – 4 Hz</td>
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</table>

Dynamic response is standardized in terms of the requirements for the frequency response nonlinearity of the device in predefined frequency range. In Table I the requirements are summarized, and it can be seen that by now there are differences in frequency ranges and values of acceptable nonlinearity.

PROBLEM STATEMENT

Since spirometry technique implies the forced expiratory maneuver when maximal airflow rate is achieved [8], standardization of frequency response nonlinearity should take into consideration dynamics of forced expiratory parameters, which is significant for the sake of diagnostics. This requirement is emphasized in papers [9, 10] when discussing the characteristics of the spirometric equipment, especially designed for LVF studies in children [11]. The literature provides information about the frequency spectrum of the forced expiratory airflow rate, which is considered as an objective criterion of the process dynamics.

In paper [12] it is determined that this frequency spectrum with amplitudes of harmonics up to 5% of the maximum were located in range $6.49 \pm 1.8$ Hz. Authors of [8] have found that the amplitude of harmonics is reduced exponentially with increasing frequency, and in the range up to 10 Hz the values are 3–5% of maximum amplitude. In [13] the bandwidth is defined in range from 0 up to 10.3 Hz. Thus all data about the frequency spectrum are rather contradictory and must be clarified.

The purpose of this paper is to define by studying the model of forced expiratory process the frequency range in which frequency response must be standardized. The airflow process during breathing is simulated using the electrical circuit analogy, and an explicit mathematical expression of airflow rate spectral density is obtained. From this expression we define the frequency range in which the dynamic characteristics of spirometers should be standardized.

THE CIRCUIT MODEL OF FORCED EXPIRATORY PROCESS

The analogy between the electric current flow and airflow can be used in the respiration process modeling. Table 2 shows the correspondence between parameters of airflow and electric circuit [14].

The considered model was used in [15, 16] for calculation of the measurement errors, optimization and control of spirometer’s dynamic characteristics, generation of test signals and spirometer metrology. It formalizes breathing in terms of electric current flow, and gives clear analogy of volumetric and velocity parameters of...
forced expiratory, giving the possibility to model various states of LVF by varying values of $R_{aw}$, $C$, $L$ and $I$.

Table 2.

<table>
<thead>
<tr>
<th>Airflow system</th>
<th>Electric circuit</th>
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<tr>
<td>$R_{aw}$ – airways resistance, Pa·sec / Liter</td>
<td>$R$ – resistance of the resistor, Ohm</td>
</tr>
<tr>
<td>$C$ – lung compliance, Liter/Pa</td>
<td>$C$ – capacitance of the capacitor, F</td>
</tr>
<tr>
<td>$I$ – intance, Pa·sec$^2$ / Liter</td>
<td>$L$ – inductance of the inductor, H</td>
</tr>
<tr>
<td>$V$ – air volume, Liter</td>
<td>$q$ – charge of capacitor, C</td>
</tr>
<tr>
<td>$p$ – pressure, Pa</td>
<td>$U$ – voltage, V</td>
</tr>
<tr>
<td>$Q$ – airflow rate, Liter / sec</td>
<td>$i$ – current, A</td>
</tr>
</tbody>
</table>

Solving (1) for the capacitor charge $q$ and current $i$, the equations for the respiration parameters of interest can be obtained:

\[
V_t = V_0 \left( \frac{1 - \alpha \cdot e^{\beta t} - \beta \cdot e^{\alpha t}}{\alpha - \beta} \right),
\]

\[
Q_t = V_0 \frac{\alpha \cdot \beta}{\alpha - \beta} \left( e^{\alpha t} - e^{\beta t} \right),
\]

where $V_0$ is forced vital capacity (FVC);

\[
\alpha, \beta = -\frac{R_{aw}}{2I} \pm \frac{R_{aw}^2}{4I^2} - \frac{1}{I \cdot C}.
\]
In normal LVF conditions parameters $\alpha$ and $\beta$ take the following values: $R_{AW} = 110 - 350 \text{ Pa}\cdot\text{sec/Liter}$, $C_L = 0,0015 - 0,003 \text{ Liter/Pa}$, $I = 1 - 17 \text{ Pa}\cdot\text{sec}^2/\text{Liter}$ [17]. It is shown in [15] that for any possible combination of $R_{AW}$, $C_L$, $I$ in normal and pathologic conditions, the values of $\alpha$ and $\beta$ are strictly real and negative since the following condition always holds true:

$$R_{AW} \geq 2\sqrt{\frac{I}{C_L}}. \quad (5)$$

**FREQUENCY CHARACTERISTICS OF AIRFLOW RATE**

To study the frequency characteristics of airflow, the Fourier spectrum of airflow rate should be defined. It is known from [15] that the time when airflow rate reaches its maximum during forced expiratory process (peak flow rate, PFR) can be defined by:

$$T_{PFR} = \frac{\ln \frac{\beta}{\alpha}}{\alpha - \beta}. \quad (6)$$

Fourier transform $F(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$ can be applied to (3) to obtain spectra of airflow rate. To facilitate this, rewrite (3) in the form

$$Q_x = Q_{max} (e^{\alpha t} - e^{\beta t}) = Q_{max} e^{\alpha t} - Q_{max} e^{\beta t}, \quad (7)$$

which is the difference of two exponential functions, and denote

$$x(t) = e^{\alpha t} \text{ for } |\alpha| > 0, \ t \geq 0, \quad (8)$$

$$y(t) = e^{\beta t} \text{ for } |\beta| > 0, \ t \geq 0. \quad (9)$$

Spectral density can be written as

$$G_{xy}(j\omega) = G_{x}(j\omega) - G_{y}(j\omega), \quad (10)$$

where $G_{x}(j\omega)$ and $G_{y}(j\omega)$ are spectral densities of exponential functions (8) and (9) respectively. It is known that Fourier transform of function $z(t) = e^{-\alpha t}$, $c > 0$, $t \geq 0$ given by $G(j\omega) = \int_{0}^{\infty} e^{-\alpha t} e^{-j\omega t} dt$ equals:

$$G_{z}(j\omega) = \frac{1}{c + j\omega} = \frac{c - j\omega}{c^2 + \omega^2} = \frac{c}{c^2 + \omega^2} - j \frac{\omega}{c^2 + \omega^2}.$$
Thus making all needed transforms and substitutions using (2) and (5) we obtain finally the complex Fourier spectrum:

\[ G_Q(j\omega) = \frac{Q_{\text{max}}}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)} \times \left\{ \left[ \alpha (\beta^2 + \omega^2) - \beta (\alpha^2 + \omega^2) \right] - j\omega (\alpha^2 - \beta^2) \right\} \]

and spectral density:

\[ P_Q(\omega) = \frac{Q_{\text{max}}}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)} \times \sqrt{\alpha (\beta^2 + \omega^2) - \beta (\alpha^2 + \omega^2)}^2 + \omega^2 (\alpha^2 - \beta^2)^2, \]

where \( Q_{\text{max}} \) is PFR at a time instant \( T_{PFR} \).

Having the expression of spectral density, it is possible to calculate it explicitly for any combination of respiration system parameters.

**RESULTS**

It is proposed to select the spectral range in which dynamic characteristics should be standardized, for the case when the spectral range of airflow rate is widest. For this case the range with harmonic components with high magnitudes should be defined at certain level of magnitudes.

To define the frequency range in which the harmonics with significant magnitudes are located, the values of \( \alpha \) and \( \beta \) should be substituted in (12). To obtain the widest frequency range of airflow rate, which corresponds to the situation with minimal \( T_{PFR} \), expression (4) should be considered. It can be seen that minimal \( T_{PFR} \) is reached when \( C_L \) and \( I \) are minimal and \( R_{AW} \) is maximal.
This case corresponds to the condition of severe LVF dysfunction, when:
\[ R_{AW} = 900 \text{ Pa-sec/Liter}, \]
\[ C_L = 0.0015 \text{ Liter/Pa}, \]
\[ I = 1 \text{ Pa-sec}^2/\text{Liter}. \]

In this case \( \alpha = -0.74 \text{ sec}^{-1} \) and \( \beta = -899.26 \text{ sec}^{-1} \).

Figure 2 shows time dependence of airflow rate as the result of simulation the respiration process using the model from Fig. 1 with formula (3) and parameters for severe LVF dysfunction. In Fig. 3 its spectral density is shown.

Using the graph from Fig. 3 it can be seen that harmonics with magnitudes larger than 2% of PFR are located in the frequency range from 0 to 70 Hz.

**DISCUSSION**

The results of defining the spectral range in which harmonics with high magnitudes are located shows, that this range obtained in our study is significantly wider than the frequency ranges reported in [8, 12]. Our result is close to the range of airflow rates frequency spectrum during cough shock (5–70 Hz [18]), which is similar to the forced expiratory process.

From our data it follows that the spirometer with frequency response normalized following the standard [7] in the frequency range of 0–15 Hz, in the considered case can measure the data with an accuracy of less than 7.5 %, which might be considered as rather high error. We can recommend from our results that if the error of forced expiratory airflow rates measurement is bounded by ± 2 %, the frequency range for standardization should be extended up to 80 Hz.

**CONCLUSIONS**

Our findings demand to normalize the dynamic characteristics of spirometers, adequate to frequency spectrum of forced expiratory airflow rates. Using an explicit mathematical expression of airflow rate spectral density, the frequency range in
which the dynamic characteristics of spirometers should be standardized is defined as 0–80 Hz. A further area of research should be focused on simulation of forced expiratory process for various combinations of $R_{aw}$, $C_L$ and $I$ in the normal LVF state and its possible violations.

SPECTRAL ANALYSIS OF AIRFLOW RATE DURING FORCED EXPIRATORY PROCESS

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Introduction: The dynamics of airflow rate using the circuit model of respiration system is considered with respect to standardization of requirements for the frequency response nonlinearity of spirometers. Dynamic response is an important characteristic of spirometers for investigation of lung ventilation function by measuring airflow rate of air during breathing. Since spirometry technique implies the forced expiratory maneuver when maximal airflow rate is achieved, standardization of frequency response nonlinearity should take into consideration dynamics of forced expiratory parameters, which is meaningful for the sake of diagnostics.

Purpose: To study the dynamic characteristics of spirometers, and development of method for analysis of airflow rate spectral density.

Methods: The analogy between the electric current flow and airflow is used to model the respiration process. It formalizes breathing in terms of electric current flow, and gives clear analogy of volumetric and velocity parameters of forced expiratory, giving the possibility to model various states of lung ventilation function. Frequency characteristics of the volumetric airflow rate are obtained using Fourier analysis of respiration parameters.

Results: An explicit mathematical expression of airflow rate spectral density is obtained and studied, and the frequency range in which the dynamic characteristics of spirometers should be standardized is defined as 0 – 80 Hz.

Conclusions: Our findings demand to normalize the dynamic characteristics of spirometers, adequate to frequency spectrum of forced expiratory airflow rates. Using an explicit mathematical expression of airflow rate spectral density, the frequency range in which the dynamic characteristics of spirometers should be standardized is defined as 0 – 80 Hz. A further area of research should be focused on simulation of forced expiratory process for various combinations of respiration parameters.

Keywords: spirometer; dynamic response; airflow rates; forced expiratory process modeling.


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